

**High-order fractional microwave-induced resistance oscillations in two-dimensional systems**S. Wiedmann,<sup>1,2</sup> G. M. Gusev,<sup>3</sup> O. E. Raichev,<sup>4</sup> A. K. Bakarov,<sup>3,\*</sup> and J. C. Portal<sup>1,2,5</sup><sup>1</sup>LNCMI-CNRS, UPR 3228, BP 166, 38042 Grenoble Cedex 9, France<sup>2</sup>INSA Toulouse, 31077 Toulouse Cedex 4, France<sup>3</sup>Instituto de Física da Universidade de São Paulo, CP 66318, CEP 05315-970 São Paulo, SP, Brazil<sup>4</sup>Institute of Semiconductor Physics, NAS of Ukraine, Prospekt Nauki 45, 03028 Kiev, Ukraine<sup>5</sup>Institut Universitaire de France, 75005 Paris, France

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We report on the observation of microwave-induced resistance oscillations associated with the fractional ratio  $n/m$  of the microwave irradiation frequency to the cyclotron frequency for  $m$  up to 8 in a two-dimensional electron system with high electron density. The features are quenched at high microwave frequencies independent of the fractional order  $m$ . We analyze temperature, power, and frequency dependencies of the magnetoresistance oscillations and discuss them in connection with existing theories.

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**I. INTRODUCTION**

In recent years, remarkable effects in two-dimensional (2D) electron systems have been discovered in the presence of microwave (MW) illumination and a classically strong transverse magnetic field  $B$ . It has been observed that the magnetic field dependence of the longitudinal resistance in a high-mobility 2D electron gas exhibits oscillations with a period determined by the ratio of the radiation frequency  $\omega$  to the cyclotron frequency  $\omega_c = eB/m^*$ , where  $m^*$  is the effective mass of electrons.<sup>1</sup> Later experiments on the samples with ultrahigh mobility have shown that for a sufficiently high radiation power the resistance minima evolve into “zero resistance states” (ZRS), where dissipative resistance vanishes.<sup>2,3</sup> Both the ZRS and the microwave-induced resistance oscillations (MIROs) have attracted much theoretical interest and several microscopic mechanisms responsible for these phenomena have been proposed. Two of these mechanisms, widely discussed in literature, produce similar results concerning the MIRO periodicity and phase. The first, “displacement” mechanism,<sup>4,5</sup> accounts for the displacement of electrons along the applied dc field under scattering-assisted microwave absorption. The second, “inelastic” mechanism,<sup>6</sup> is associated with a microwave-generated nonequilibrium oscillatory component of the isotropic part of electron distribution function. The MIROs can also appear owing to the “photovoltaic” mechanism<sup>7</sup> describing combined action of the microwave and dc fields on both temporal and angular harmonics of the distribution function. Finally, it has been predicted that microwave excitation of the second angular harmonic of the distribution function, which is referred to as the “quadrupole” mechanism,<sup>7</sup> should lead to an oscillatory contribution to the transverse (Hall) resistivity. Among these mechanisms, the inelastic one is thought to play the dominant role in experiments at low temperatures  $T$  and microwave powers because its contribution is parametrically large in comparison to contributions of the other mechanisms. In particular, the inelastic mechanism overcomes the displacement one by a large factor  $\tau_{\text{in}}/\tau_q \propto T^{-2}$ , where  $\tau_{\text{in}}$  and  $\tau_q$  are the inelastic-scattering time and quantum lifetime, respectively. It has been found that the inelastic mechanism satis-

factory explains the power and temperature dependence of resistivity observed in earlier experiments and in more recent experiments both on single-subband<sup>8</sup> and two-subband 2D systems.<sup>9</sup>

With increasing microwave power, notable MIRO features associated with fractional ratios

$$\epsilon \equiv \omega/\omega_c = n/m, \quad (1)$$

where  $n$  and  $m$  are integers, have been observed.<sup>8,10-12</sup> While all the resonances described by Eq. (1) originate from commensurability of the cyclotron and microwave frequencies, the fractional MIROs (FMIROs) corresponding to  $m \geq 2$  require multiphoton absorption processes which appear in the strongly nonlinear regime and can proceed in two ways. The first is a simultaneous absorption of several photons (with virtual intermediate states)<sup>13,14</sup> and the second is a stepwise absorption of single photons.<sup>15</sup> The latter process contributes to resistivity owing to incomplete relaxation of the electron system between the absorption events. The MIRO theories accounting for these processes<sup>13-15</sup> demonstrate progressive appearance of the oscillation peaks with increasing fractional denominator  $m$  as the microwave power increases, which is in agreement with experimental findings.<sup>8,10-12</sup> The experiment<sup>8</sup> shows the FMIRO features up to  $m=4$  ( $\epsilon = 1/4$ ).

In this paper we report the observation of oscillatory features which are likely attributed to higher-order (up to  $\epsilon = 1/8$ ) FMIROs. These oscillations are more pronounced at high temperatures ( $T \approx 6.5$  K), when their observation is not hindered by the presence of Shubnikov-de Haas (SdH) oscillations. We analyzed temperature, power, and frequency dependencies of the magnetoresistance oscillations. For the microwave powers used in our experiment, the simultaneous absorption of more than two photons is found to be a rare event. Therefore, a more plausible explanation for the high-order FMIROs is the stepwise absorption of single photons.<sup>15</sup> However, this mechanism also fails to describe observation of high-order fractions because of energy conservation restrictions, when the gap between the Landau levels exceeds the photon energy. Moreover, we have found that the amplitude of FMIROs for *all* fractions dramatically decreases at

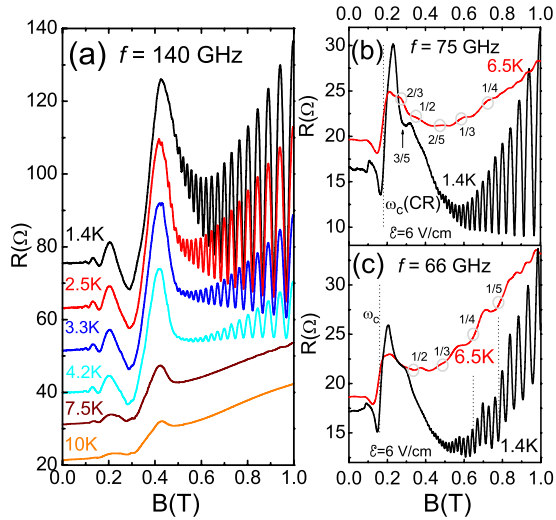


FIG. 1. (Color online) (a) Temperature-dependent magnetoresistance  $R_{xx}$  of MIROs under microwave irradiation of 140 GHz. The curves (except the one for 10 K) are shifted for clarity. (b) and (c) FMIROs for 75 and 66 GHz at  $T=1.4$  and  $T=6.5$  K. CR (dotted line) and FMIRO features are marked with circles for corresponding  $\epsilon$ .

high microwave frequencies ( $f \geq 85$  GHz). This behavior qualitatively disagrees with the stepwise absorption model,<sup>15</sup> which predicts that the threshold frequency strongly depends on the fractional value. We believe that this FMIRO quenching effect is very similar to integer MIRO damping which occurs at higher frequencies.<sup>16,17</sup>

## II. EXPERIMENT

The measurements have been performed in GaAs quantum wells (QWs) with well widths of 14 nm in the presence of microwave irradiation in the range from 32 to 100 GHz with different intensities of radiation. In contrast to the previous studies<sup>8,10,11</sup> with low electron density ( $< 3.6 \times 10^{11} \text{ cm}^{-2}$ ) and ultrahigh mobility ( $\sim 2 \times 10^7 \text{ cm}^2/\text{V s}$ ), we use high-density samples from different wafers with moderate mobilities  $< 1.8 \times 10^6 \text{ cm}^2/\text{V s}$ . All the data presented below are for a sample with a carrier concentration of  $7.8 \times 10^{11} \text{ cm}^{-2}$  and a mobility of  $1.2 \times 10^6 \text{ cm}^2/\text{V s}$ . The sample was mounted in a VTI cryostat with a waveguide to deliver radiation down to the sample. The resistance  $R=R_{xx}$  was measured by using a standard low-frequency lock-in technique (13 Hz) under continuous MW illumination. Whereas previous experiments<sup>8,10-12</sup> have been carried out at low temperature ( $T \sim 1$  K), we extended our studies up to 10 K. Figure 1(a) presents temperature dependence of MIRO from 1.4 to 10 K for a MW irradiation of 140 GHz with 0 dB (highest intensity). For 1.4 K, we see the cyclotron resonance (CR) peak ( $n=1$ ) as well as integer MIROs with  $n=2$  and 3. No ZRS are observed in these samples. With increasing temperature, the MIRO peaks  $n=1$  and 2 survive up to 10 K. For low frequencies ( $f < 40$  GHz), the CR-peak amplitude starts to decrease rapidly and vanishes for 35 GHz. This is attributed to the strong suppression of the Landau quantization by

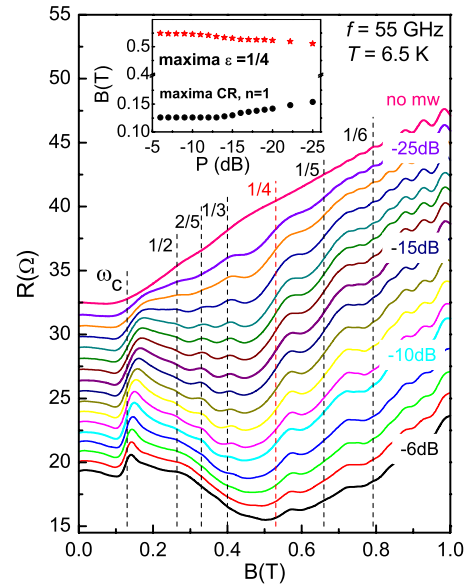


FIG. 2. (Color online) (a) Power dependence for 55 GHz from  $-6$  to  $-25$  dB for  $f=55$  GHz at 6.5 K. The curves (except the one for  $-6$  dB) are shifted for clarity. Top trace (no MW) is the magnetoresistance without microwave irradiation. Positions of the maxima (CR and FMIROs up to  $\epsilon=1/6$ ) are marked by dashed lines. The inset shows magnetic field dependence of these maxima with decreasing MW power.

disorder (i.e., to the exponential smallness of the Dingle factors) in the region of  $B$  corresponding to the CR.

Figures 1(b) and 1(c) illustrate magnetoresistance measurements for 75 and 66 GHz at 1.4 and 6.5 K, respectively. At 1.4 K we clearly see only one FMIRO feature for each frequency and for 66 GHz we also observe a kind of modulation at the positions where FMIROs are precised at a temperature of 6.5 K. For 6.5 K, the observations at both 75 and 66 GHz reveal clearly pronounced FMIROs. FMIRO positions  $n/m$  are marked according to Eq. (1) with circles for each fractional feature. In order to reproduce these experimental results, we performed power-dependent and frequency-dependent measurements up to a MW frequency of 100 GHz.

We now focus on power dependence of FMIROs at 6.5 K, where we find the best pronounced features and present our power-dependent measurement for 55 GHz where we observe more fractional features up to  $\epsilon=1/6$  ( $\epsilon=1/7$  is already superimposed by SdH oscillations and not marked in Fig. 2). In Fig. 2 this power dependence is presented for MW attenuations from  $-6$  to  $-25$  dB. For  $-6$  dB (bottom trace), FMIROs  $\epsilon=1/4$  and  $\epsilon=1/5$  indicate distinctive local minima and maxima.

With decreasing microwave power, the picture is improved as well as new FMIROs in lower magnetic field start to appear ( $\epsilon=1/3$ ,  $\epsilon=2/5$ , and  $\epsilon=1/2$ ) for  $-10$  dB. The amplitude of these low-field FMIROs increases if one further decreases MW power. Looking closer at different FMIROs, it can be noticed that down to  $-25$  dB, the amplitude of  $\epsilon=1/5$  is already suppressed whereas  $\epsilon=1/4$  still exhibits maxima and minima. By plotting the position of maxima of CR and FMIRO  $\epsilon=1/4$  (dotted lines) in magnetic field as a

function of power, see the inset to Fig. 2, we find that for decreasing MW power the FMIRO  $\epsilon=1/4$  maxima are shifted to lower magnetic field in contrast to the evolution of the CR peak where the maxima shift to a higher magnetic field.<sup>6</sup> The top trace in Fig. 2 is without MW irradiation. Notice that for  $5 < T < 15$  K we see magnetophonon resonances owing to interaction of electrons with acoustic phonons.<sup>18</sup>

### III. DISCUSSION OF THE RESULTS

We now discuss the observed frequency, power, and temperature dependence of FMIROs in our samples. The observation of high-order fractional features can distinguish between the multiphoton model<sup>14</sup> and the stepwise single-photon mechanism.<sup>15</sup> In fact, absorption of  $m > 2$  photons requires a very high microwave power because the corresponding contributions into the photoresistance<sup>7,14</sup> should decrease as  $(W_{\pm})^m$ , where the dimensionless parameters  $W_{\pm}$  (+ and - stand for two components of microwave radiation with different circular polarization) are defined as

$$W_{\pm} = \frac{\tau_q}{\tau_{tr}} \left[ \frac{e\mathcal{E}v_F}{\hbar\omega(\omega_c \pm \omega)} \right]^2, \quad (2)$$

and depend on the MW electric field  $\mathcal{E}$ , Fermi velocity  $v_F$ , and on the ratio of quantum lifetime  $\tau_q$  to transport time  $\tau_{tr}$ . This ratio is usually smaller than 0.1 and is estimated as 1/15 for our samples. For the MW powers used in our experiments [we estimate  $\mathcal{E}=1-10$  V/cm (see Fig. 2) by comparing the suppression of SdH oscillations with the dc electric field due to the heating effect<sup>9</sup>] and reported in the previous studies,<sup>8,10,11</sup> the parameters  $(W_{\pm})^m$  are vanishingly small under FMIRO conditions  $\omega_c = m\omega$ , except for the case  $m=2$  at the highest powers. The fractional oscillations near  $\epsilon=1/2$  and  $3/2$  observed in previous publications, in principle, can be explained by the two-photon process.<sup>11,12</sup> However, the features near  $\epsilon=1/3$ ,  $1/4$ ,  $2/3$ , and  $2/5$  are hardly ascribed to this model. From this point of view, FMIRO due to the process of stepwise absorption of the several photons seems to be more suitable. Within the framework of this model,<sup>15</sup> the intensity of the fractional resonances with denominator  $m$  is proportional to the  $m$ th power of the squared Dingle factor,  $\exp(-2m\alpha)$ , where  $\alpha = \pi/\omega_c\tau_q$ . This explains why the fractional features are absent for the low-field side of the cyclotron resonance (the disappearance of integer MIRO and a rapid decrease in the CR-peak amplitude with lowering MW frequency is of the same origin). However, it should be also expected that FMIROs damp exponentially faster than the integer oscillations due to a decrease in the quantum lifetime with increasing temperature.<sup>19</sup> In contrast, we observe that the FMIRO features weaken and eventually disappear at the same temperature as the integer MIRO features. It may be argued that this difference is not essential after a transition from overlapped to separated Landau levels, where  $\exp(-\alpha)$  is of the order unity, and many oscillatory harmonics contribute into the density of states. Nevertheless, the fractional features in the stepwise absorption model are in any case more sensitive to broadening or damping factors than the integer features.

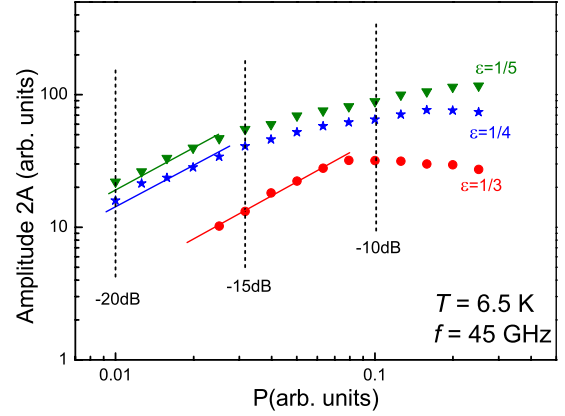


FIG. 3. (Color online) Analysis of the power dependence of amplitudes for CR and FMIROs  $\epsilon=1/3$ ,  $1/4$ , and  $1/5$  for 45 GHz. The amplitude  $2A$  (peak to peak) is extracted from the derivative of  $R$  as a function of magnetic field. The straight lines correspond to a linear power dependence.

Let us consider power and frequency dependence as well as the threshold for fractional MIROs. It is seen already from Fig. 2 that FMIROs are sensitive to MW power. Figure 3 presents the power dependence of the amplitude  $2A$ , extracted from the derivative  $dR/dB$ , for 45 GHz at 6.5 K for FMIROs  $\epsilon=1/3$ ,  $1/4$ , and  $1/5$ . The FMIRO amplitudes show a dependence close to linear for weaker MW powers while for higher powers the dependence becomes sublinear and weakly dependent on the power. This behavior is similar to that for integer MIRO and is consistent with the stepwise photon-absorption mechanism.<sup>15</sup> The transition to a weaker power dependence is attributed to the saturation effect and confirms the importance of the effect of microwaves on the electron distribution function, which is the basic feature of the inelastic mechanism of photoresistance.<sup>6</sup> It is worth noting that in the region of MW power where we observe FMIROs the power dependence of the CR-peak amplitude is already sublinear, which means that the saturation regime occurs earlier at  $\omega \approx \omega_c$ , in agreement with the theory.<sup>6</sup>

The measured magnetoresistance as a function of the magnetic field for different frequencies up to 90 GHz at 6.5 K is shown in Fig. 4(a). All observed FMIROs  $\epsilon=n/m$  for the chosen frequencies in Fig. 4(a) are also presented in Table I and in Fig. 5 with different symbols. Apart from FMIROs with  $n=1$  and  $m=2, 3, \dots$ , which exhibit clear features of maxima and minima, we also see weak features at  $\epsilon=2/3$ ,  $2/5$ , and  $2/7$ . No FMIROs for  $\omega < \omega_c$  ( $m < n$ ) are observed. We find experimentally a threshold frequency at  $f_{th} \approx 90$  GHz, which is higher than in the previous studies.

The results of theoretical calculations accounting for both inelastic and displacement mechanisms of photoresistance under both simultaneous and stepwise multiphoton processes<sup>13-15</sup> are presented in Fig. 4(b). The calculation are done using the self-consistent Born approximation (SCBA), with  $\tau_q=3$  ps and  $\tau_{in}/\tau_{tr}=2$ , according to our estimates for these parameters at  $T=6.5$  K. While the theoretical curves reproduce general behavior of the magnetoresistance and show low-order FMIROs, they do not show high-order FMIROs. The reason for this is the following. For observa-

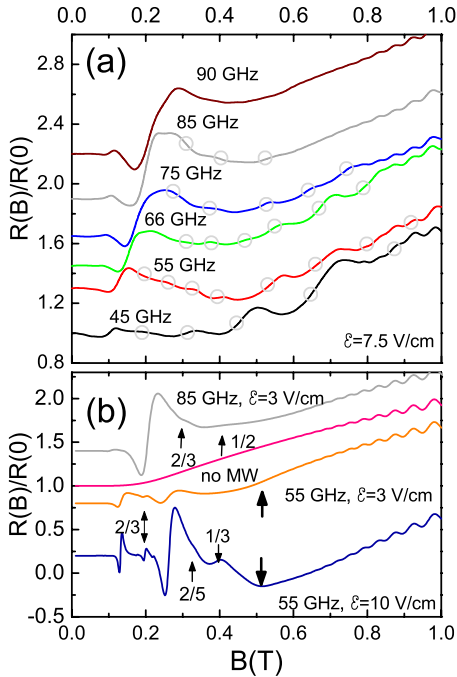


FIG. 4. (Color online) (a) Experimental and (b) theoretical magnetoresistance for different frequencies at 6.5 K (electric field  $\mathcal{E} = 7.5$  V/cm). The curves are shifted for clarity. All FMIROs are marked by circles and the corresponding values are listed in Table I. For 90 GHz all the experimental FMIRO features disappear. The theoretical plots reveal several FMIROs up to  $\epsilon = 1/3$  at  $\mathcal{E} = 10$  V/cm but show no high-order FMIROs.

tion of the commensurability resonances [see Eq. (1)] associated with stepwise single-photon transitions between the adjacent Landau levels ( $n=1$ ) in the regime of separated Landau levels ( $\alpha < 2$  according to SCBA) one should satisfy the relation

$$\hbar\omega_c - 2\Gamma < \hbar\omega < \hbar\omega_c + 2\Gamma, \quad (3)$$

where  $\Gamma = \hbar\omega_c [\arccos(1-\alpha) + \sqrt{(2-\alpha)\alpha}]/2\pi$  is the half-width of the Landau level in the SCBA. While the upper limit for  $\omega$  in Eq. (3) is not essential in the range of frequencies and magnetic fields we study, the lower limit imposes severe constraints for observation of high-order FMIRO, as illustrated in Fig. 5.

TABLE I. Fractional microwave-induced resistance oscillations for all frequencies in Fig. 4(a) at 6.5 K. The estimated electric field for all experimental traces is  $\mathcal{E} = 7.5$  V/cm.

Frequency (GHz)	FMIROs $\epsilon = n/m$
45	1/2, 1/3, 1/4, 1/6, 1/8 <sup>a</sup>
55	2/3, 1/2, 2/5, 1/3, 1/4, 1/5, 1/6, 1/7 <sup>a</sup>
66	1/2, 2/5, 1/3, 2/7, 1/4, 1/5
75	2/3, 1/2, 1/3, 2/7, 1/4
85	2/3, 1/2, 2/5

<sup>a</sup>Superimposed by SdH oscillations.

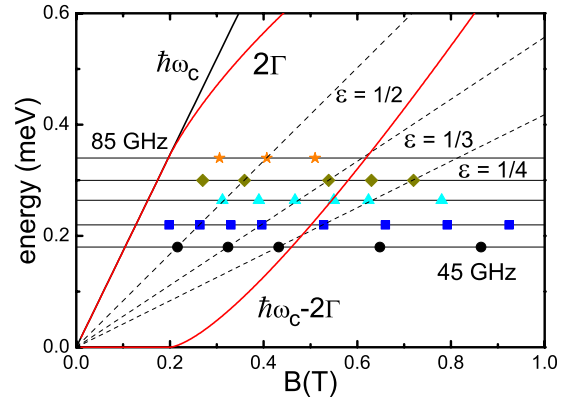


FIG. 5. (Color online) Fractions of cyclotron energy, photon energies, and the threshold energy  $\hbar\omega_c - 2\Gamma$  as functions of the magnetic field. The positions of the observed fractional features are marked by different symbols for each MW frequency. FMIRO features beyond the border line cannot be described within the mechanism of stepwise multiphoton absorption.

The intersection of the photon energy  $\hbar\omega$  with the dotted lines marks position of FMIRO at  $\epsilon = 1/m$  while the intersection of the dotted lines with the border line  $\hbar\omega_c - 2\Gamma$  shows the threshold energies for corresponding fractions. The FMIRO in our theoretical calculations appear below this threshold while at the border  $\hbar\omega = \hbar\omega_c - 2\Gamma$  the magnetoresistance plots have bumplike features clearly visible at high MW power and shown by broad arrows in Fig. 4(b). Note that the quantum lifetime, and, therefore, the half-width depends on the temperature, thus the high-order FMIROs are favored at high  $T$ . As seen, several features with high order observed at lower  $\omega$  are not allowed by Eq. (3), especially those for 45 and 55 GHz which occur roughly at the same magnetic field. On the other side, the FMIRO with  $\epsilon = 1/2$  is allowed at frequencies above 85 GHz, which also disagrees with our observations. It is worth noting that recently it has been demonstrated in experiments that the integer MIROs are quenched at frequencies above 200 GHz and practically disappear at higher frequencies.<sup>16</sup> Note that in Ref. 17 the mobility is two times higher than in Ref. 16 which causes the MIRO observation at 240 GHz. The microscopic mechanism of this quenching effect is still unclear. We believe that a common theoretical approach to explain the damping effect of both fractional and integer MIROs is necessary.

#### IV. CONCLUSION

In summary, we observe fractional MIROs with ratios  $\epsilon = \omega/\omega_c = n/m$  in a high-density two-dimensional electron system for different frequencies and relatively high temperatures when SdH oscillations are suppressed. Fractional MIROs have a similar temperature dependence to integer MIRO, which may indicate that FMIROs are dominated by the inelastic mechanism under multiphoton absorption. The transition to sublinear power dependence of FMIRO amplitude with increasing microwave power, which we attribute to saturation effect, is another argument in favor of the inelastic mechanism.<sup>6</sup> We have analyzed two competing models for



FMIRO, a simultaneous multiphoton absorption and a stepwise absorption of several photons, in application to our data. The multiphoton model fails to explain the high-order fractional features because the microwave power is not sufficient to generate such multiphoton processes. Concerning the stepwise absorption model, both the existence of high-order FMIRO in the absence of single-photon transitions between Landau levels and the independence of the threshold frequency for FMIRO quenching of the number of fraction disagree with the predictions of this model. Therefore we

believe that the high-order fractional features require another explanation. More experimental work and theoretical effort has to be done in order to obtain a complete understanding of this FMIRO phenomenon.

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<sup>1</sup>M. A. Zudov, R. R. Du, J. A. Simmons, and J. L. Reno, *Phys. Rev. B* **64**, 201311(R) (2001).

<sup>2</sup>R. G. Mani, J. H. Smet, K. von Klitzing, V. Narayanamurti, W. B. Johnson, and V. Umansky, *Nature (London)* **420**, 646 (2002).

<sup>3</sup>M. A. Zudov, R. R. Du, L. N. Pfeiffer, and K. W. West, *Phys. Rev. Lett.* **90**, 046807 (2003).

<sup>4</sup>V. I. Ryzhii, *Fiz. Tverd. Tela (Leningrad)* **11**, 2577 (1969) [*Sov. Phys. Solid State* **11**, 2078 (1970)]; V. I. Ryzhii, R. A. Surris, and B. S. Shchamkhalova, *Fiz. Tekh. Poluprovodn.* **20**, 2078 (1986) [*Sov. Phys. Semicond.* **20**, 1299 (1986)].

<sup>5</sup>A. C. Durst, S. Sachdev, N. Read, and S. M. Girvin, *Phys. Rev. Lett.* **91**, 086803 (2003).

<sup>6</sup>I. A. Dmitriev, M. G. Vavilov, I. L. Aleiner, A. D. Mirlin, and D. G. Polyakov, *Phys. Rev. B* **71**, 115316 (2005).

<sup>7</sup>I. A. Dmitriev, A. D. Mirlin, and D. G. Polyakov, *Phys. Rev. B* **75**, 245320 (2007).

<sup>8</sup>S. I. Dorozhkin, J. H. Smet, K. von Klitzing, L. N. Pfeiffer, and K. W. West, *JETP Lett.* **86**, 543 (2007).

<sup>9</sup>S. Wiedmann, G. M. Gusev, O. E. Raichev, T. E. Lamas, A. K. Bakarov, and J. C. Portal *Phys. Rev. B* **78**, 121301(R) (2008).

<sup>10</sup>S. I. Dorozhkin, J. H. Smet, V. Umansky, and K. von Klitzing, *Phys. Rev. B* **71**, 201306(R) (2005).

<sup>11</sup>M. A. Zudov, R. R. Du, L. N. Pfeiffer, and K. W. West, *Phys. Rev. B* **73**, 041303(R) (2006).

<sup>12</sup>R. G. Mani, J. H. Smet, K. von Klitzing, V. Narayanamurti, W. B. Johnson, and V. Umansky, *Phys. Rev. Lett.* **92**, 146801 (2004).

<sup>13</sup>X. L. Lei and S. Y. Liu, *Appl. Phys. Lett.* **88**, 212109 (2006).

<sup>14</sup>I. A. Dmitriev, A. D. Mirlin, and D. G. Polyakov, *Phys. Rev. Lett.* **99**, 206805 (2007).

<sup>15</sup>I. V. Pechenezhskii, S. I. Dorozhkin, and I. A. Dmitriev, *JETP Lett.* **85**, 86 (2007).

<sup>16</sup>S. A. Studenikin, A. S. Sachrajda, J. A. Gupta, Z. R. Wasilewski, O. M. Fedorych, M. Byszewski, D. K. Maude, M. Potemski, M. Hilke, K. W. West, and L. N. Pfeiffer, *Phys. Rev. B* **76**, 165321 (2007).

<sup>17</sup>R. G. Mani, *Appl. Phys. Lett.* **92**, 102107 (2008).

<sup>18</sup>M. A. Zudov, I. V. Ponomarev, A. L. Efros, R. R. Du, J. A. Simmons, and J. L. Reno, *Phys. Rev. Lett.* **86**, 3614 (2001).

<sup>19</sup>N. C. Mamani, G. M. Gusev, T. E. Lamas, A. K. Bakarov, and O. E. Raichev, *Phys. Rev. B* **77**, 205327 (2008).